Building Mathematical Comprehension

Using Literacy Strategies to Make Meaning

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Table of Contents

Chapter 1

Comprehension Strategies for Mathematics

Teachers shoulder an enormous responsibility. Our society relies on them to provide a solid foundation for the future of our young people. What a sobering thought! The instructional decisions they make daily impact the quality of the educational underpinnings of their students. The success of each student rests upon these foundations.

Whether gathering ideas from educational materials, collaborating with other teachers, or reflecting on past experiences, teachers make difficult instructional decisions. During the many routine and sometimes mundane tasks required of them, most teachers experience a background "hum" that persists throughout the day. It's a hum that keeps them thinking, questioning, and seeking the most effective learning environment for their students. From the "I wish I had" and the "that worked well" to the "I bet that would be effective," teachers often struggle to develop instructional repertoires that most successfully meet the academic needs of their students. In recent years, schools and educators have received more than their share of criticism. Just as tides come and go, beliefs about what constitutes "good" teaching seem to shift.

Concerns about Mathematical Achievement in the United States

In 1997, because of concern about the reading ability of students in the United States, Congress established a national panel to assess the effectiveness of various approaches used to teach children to read. Following two years of reviewing research and conducting public hearings, the National Reading Panel (NRP) issued its report (National Reading Panel 2000). While the contents of the report were controversial, the report did generate increased awareness of the critical components of teaching reading and spurred increased attention and funding for reading instruction. The National Reading Panel found that in addition to teaching skills aimed at decoding words and reading fluency, explicit instruction focusing on a combination of comprehension techniques was effective and should be a part of reading instruction. Based on the findings of the NRP, school systems throughout the country began to provide professional development to their teachers aimed at improving reading instruction.

After years of focus on reading achievement, the national attention began to shift. In light of concerns about the performance of United States students on international tests and the effects of the emergence of a new global economy, worries about mathematics achievement entered the national psyche.

Global Achievement Gap in Mathematics

Due to economic, social, political and technological changes, concern developed over the perception of a global achievement gap—a gap between what our students are taught as compared to what they need to be successful in the world today (Wagner 2008).

Multinational corporations can now pick and choose from global labor markets. Can our work force compete with those of other nations? How does the United States stack up in comparison with other work forces? To see exactly how students in the United States perform on mathematics assessments compared to other nations, we can study data from two international assessments.

The Organisation for Economic Co-operation and Development (OECD) administers the Programme for International Student Assessment (PISA) every three years to measure the achievement of fifteen-year-old students in mathematics, reading, and scientific literacy. OECD (2010) defines mathematical literacy as:

> **"…an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen."**

In other words, students should be able to put mathematics to a functional use. They should have the "ability to analyze, reason, and communicate ideas effectively as they pose, formulate, solve, and interpret solutions to mathematical problems in a variety of situations or contexts" (OECD 2006).

The 2009 PISA scores in mathematics literacy for the United States students (Fleischman et al. 2010) were disappointing. Of the 64 countries and education systems participating in the testing, 23 had higher average scores than those of the United States. Furthermore, only 27 percent of students from the United States scored at or above Level 4 proficiency. At Level 4, students can complete "higher order tasks such as solving problems that involve visual and spatial reasoning in unfamiliar contexts and carrying out sequential processes" (Fleischman et al. 2010). Of the participating countries and education systems, 22 had a higher percentage of students scoring at Level 4 or above. In fact, the scores in 2009 were slightly better than those in 2006, but were not statistically different from those in 2003. Over six years, there had been virtually no improvement in the mathematical literacy scores of students in the United States.

In addition to the PISA results, the Trends in International Mathematics and Science Study (TIMSS) compares the mathematical skills of U. S. fourth- and eighth-graders to students in other countries. In 2007, the average score of our fourth-graders was higher than the average score of fourth-graders in 23 countries, but *lower* than the average score of *eight* countries and about the same as the average score for four countries. Only 10 percent of our fourth-graders scored in the advanced level, compared to 41 percent of the fourth-graders in Singapore (National Center for Education Statistics 2010).

The results were similar for U.S. eighth-grade students. Their average score was higher than those of 37 countries, *lower* than that of *five* countries, and about the same as that of five countries. Only 6 percent of U.S. eighth-graders scored at the advanced level compared to 45 percent of eighth-graders in Chinese Taipei. Clearly, the scores of the United States eighth-graders were far from those of the top performers.

The lackluster scores of our students on these assessments are surprising considering the prosperity of the United States relative to the other participating nations. They motivate us to re-examine our teaching practices. Just as the teachers described at the beginning of the chapter are searching for instructional strategies to enable their students to be successful nationwide, people both within and outside the teaching field are questioning existing teaching practices and searching for the most effective means of instilling our young people with the mathematics background they need for the twenty-first century.

Instructional Strategies for Student Achievement

All teachers have unique interests and concerns as they plan for instruction—their teaching strengths vary, as do their classes. Most teachers are forever searching for an overall set of instructional strategies to help them bring out the very best in their students. What "magic" strategies work for all subject areas and for all students?

It is not surprising that this pursuit has so often focused on methods for teaching reading. After all, reading and language are so inextricably linked to thinking. If students become proficient readers—skillful at comprehending text—surely they can be successful academically. Sometimes this proves to be true. Effective reading instruction does promote overall academic success for a portion of learners.

Oddly, even when reading strategies are seen as keys to success for students, they are rarely taught in subjects other than reading. Despite this, some students are able to take their knowledge of reading strategies and apply it successfully throughout the curriculum.

Unfortunately, many students continue to struggle with content area comprehension, failing to recognize that the strategies they learned to help them with their reading comprehension can also be applied to increase their understanding in other subject areas.

Though most educators recognize that reading ability is crucial to success in any subject area, including mathematics, the instructional strategies taught are most commonly partitioned by subject area. Sometimes this occurs because teachers specialize in only one or two subjects. More often, in elementary schools, homeroom teachers use very different strategies when teaching multiple subjects. The lack of consistency in instruction is puzzling. Instructional strategies that teachers use so successfully in reading are often abandoned when they teach mathematics and other content areas. Even when employing similar strategies, the terminology differs. So, even if students are expected to use similar approaches to understanding mathematics, the lack of consistent vocabulary creates confusion and limits the effectiveness of these approaches. As a consequence, student comprehension and understanding suffer.

Reading and Mathematics Connections

In *Comprehending Math: Adapting Reading Strategies to Teach Mathematics K–6,* Hyde (2006) describes how thinking, language, and mathematics are braided together into a "tightly knit entity like a rope that is stronger than the individual strands." Reading is closely linked to thinking and language, and so is mathematics. The three components are inseparable, mutually supportive, and necessary. In reading, students use decoding skills, and then go a step further to construct meaning by interacting with the text. In mathematics, students need to be encouraged to use the same strategies as they construct mathematical meaning (Sammons 2009). In fact, many of the characteristics of good readers are found in good mathematicians (Minton 2007). See figure 1.1 for a side-by-side comparison.

Fig. 1.1. Similarities Between Good Readers and Good Mathematicians

Characteristics of Good Readers	Characteristics of Good Mathematicians
They call upon their prior knowledge to make meaning from text.	They call upon prior knowledge to understand concepts and solve problems.
They are fluent readers.	They are procedurally fluent.
They have a mental image of what they are reading.	They create multiple representations of mathematics concepts and problems.
They use multiple strategies to understand and interpret text.	They use multiple strategies to understand concepts and solve problems.
They monitor their understanding as they read.	They monitor their understanding as they solve problems.
They can clearly explain their interpretation of the text to others.	They can clearly explain their mathematical thinking to others.

(Adapted from Minton 2007)

Because of these similarities between reading and mathematics, it is only logical that strategies employed to increase comprehension in one would be equally as effective at increasing comprehension in the other. Just as teachers are adapting the Guided Reading instructional framework to create the Guided Math framework (Sammons 2009), many teachers are now adapting literacy strategies for mathematics instruction. As Ellin Keene asked in the foreword to Hyde's *Comprehending Math,* "…if we ask kids to construct meaning in reading, wouldn't we ask them to do the same in math?" (Hyde 2006).

Reading and Mathematics Comprehension

Reading in its truest sense requires comprehension, construction of meaning from the written language. In this sense, reading comprehension goes beyond simply decoding words or reading fluently. It is the dynamic process of taking information from the written text, interacting with it, and then using it in a way that demonstrates the reader understands the information. In fact, it may accurately be said that reading is thinking (Hyde 2006).

Good readers are capable of acting on, responding to, or transforming the information that is presented in written text. They are able to go beyond a simple literal recall or retelling. According to the transformational model of reading comprehension, the reader uses his or her own thought processes and prior knowledge, along with the information from the text, to transform it in some way (Brassell and Rasinski 2008), thereby creating meaning.

Just as good readers create meaning for understanding, mathematicians create meaning as they process mathematical concepts and solve problems. Understanding the relationship between the two disciplines can provide teachers with valuable insights to help students more successfully understand the mathematics they encounter in school and in the real world.

Skillful reading requires readers to be active, as does understanding mathematical ideas. Good readers are actively constructing meaning—using what they know about the content of the text within the given context. They use their knowledge of text structure and word meaning as they draw inferences from the words they read (Hyde 2006; Owocki 2003). The same is true for mathematics—in fact, doubly so. Mathematics requires not only the construction of meaning related to mathematical concepts, but also comprehension of the written text that is so often required for problem-solving tasks.

What kinds of knowledge do readers draw upon as they read? According to Owocki (2003), this knowledge includes:

- knowledge about text content
- knowledge about text structure
- pragmatic knowledge
- knowledge about the social/situational context

Knowledge About Content

Readers who are familiar with the general subject of the text they are reading are much more likely to be able to construct meaning from that text. Familiarity with the content provides a foundation for both understanding and for building new knowledge. Hence, children with little background knowledge to build upon often struggle with comprehension. Even for those who have excellent decoding skills and read fluently, meaning eludes them. Consequently, these students frequently become frustrated. What joy is there in trying to "read" something that is incomprehensible? Anyone who has tried to read a technical text about an unfamiliar subject can empathize with the lack of motivation to read that is too often evident in these students.

When students are introduced to new mathematical concepts, the same premise holds true. Students who have some foundation to build upon can make connections and create an understanding of the concept more readily than students who lack the background knowledge. Just as good readers apply existing knowledge or schema to new information, mathematicians must apply their existing knowledge to mathematical concepts or problems to construct meaning (Minton 2007). Teachers can help students in these efforts by providing opportunities for them to build background knowledge to serve as a foundation for the new concept.

Even students who have ample background knowledge can benefit from thinking about and discussing foundational content prior to their reading or math work. Previewing spurs students to begin tapping their reservoir of prior knowledge, so that it's easier to construct meaning from the words they are decoding or the math concepts they are solving.

Knowledge About Structure

Accomplished readers are knowledgeable about the structures of the texts they read and use that knowledge to build meaning. The structure of narrative texts differs considerably from that of nonfiction texts. Immersed in a narrative, a reader expects to be introduced to characters, to identify the setting, and to discover the plot as it unfolds.

When studying a nonfiction text, on the other hand, a reader expects to encounter an organizational structure that helps him or her make sense of the content. A biography may be organized in chronological order. Readers come to expect the story of someone's life to unfold in a logical sequence of events. News stories usually begin with the most important points and then move on to the details. In recipes, the ingredients are listed first, followed by step-by-step directions.

Similarly, the study of mathematics is based on structures. Students benefit from knowing about the structure of word problems—considered by some to be a unique genre. Word problems most often have the introductory information first, then any factual information needed, followed by the main idea—what students need to determine—at the end of the problem. In addition, the discipline itself is replete with structures and frameworks on which mathematical concepts are constructed. For instance, one of the first mathematical structures students learn is that of place value. It is difficult for young learners to understand the concepts of addition or subtraction with regrouping if they do not comprehend place value.

An awareness of both textual and mathematical structures provides scaffolding for students when reading or working with mathematics. When students become aware of both the textual structures of literary genres and the mathematical structures they encounter, they have at their disposal a toolbox of resources for constructing meaning.

Pragmatic Knowledge

In addition to the content knowledge readers use to construct meaning, they also bring a wealth of socio-cultural, or *pragmatic*, knowledge gleaned through years of interactions with others. When filtered through these diverse perspectives, the constructed meanings of texts can vary dramatically. In the same way, students bring their own understandings and experiences into play as they construct mathematical meaning. Many young people have acquired problem-solving strategies outside of school. They call upon these earlier experiences as they tackle mathematical tasks within the classroom. Teachers who allow students a degree of latitude as they solve problems will be rewarded with insight into the pragmatic knowledge of their students. By respecting students' unique attempts at problem solving, teachers increase students' confidence and their willingness to take risks. As students share their strategies, the strategic resources of the entire class are broadened.

When students become aware that their interpretations of a text or strategies for solving problems vary because of the unique pragmatic knowledge that each of them brings to the task, they become aware of the limits of their perspectives, even as their perspectives are expanded.

Knowledge About the Social/Situational Context

Finally, readers are influenced by the situations in which their reading occurs. Reading a poem for the beauty of its language is quite different from reading it when assigned to analyze its meaning. The purpose or goals for reading set by a teacher in a reading assignment can shade the meaning-making of students. Likewise, students reading a mathematics word problem in search of key words may construct a very different interpretation of the problem than students attempting to visualize the problem based on their past experiences and the mathematical connections they recognize.

How teachers frame their expectations and goals for students, whether in reading or mathematics, has an enormous impact on the meanings constructed by their students. Teachers should always be aware of how the goals they set for their students influence how students interpret text or mathematical tasks.

The individual goals and beliefs students have instilled in them both at home and in school are also closely related to their mathematics performance. When students believe that their efforts determine their achievement rather than just their innate ability, they are more persistent in their mathematics learning and, thus, are more successful. The National Research Council (2001) in its study *Adding It Up: Helping Children Learn Mathematics*, referred to a positive attitude toward mathematics as productive disposition. Productive disposition is "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics." Because of its importance, it is identified as one of the five strands of mathematical proficiency.

Unfortunately, many students bring an overall aversion to math that is reflected in our popular culture. Boaler (2008) refers to a TV episode of *The Simpsons* in which Bart returns his math textbook to his teacher at the end of the school year unused, still in its original wrapping. Likewise, Mattel® toys developed a line of "speaking" Barbie dolls that exclaimed, "Math class is tough." Whether the preconceptions some students bring to their study of mathematics are positive or negative, it can only serve to enrich their understanding if they become aware that many of the preconceptions people hold about mathematics are, in fact, misconceptions.

There is a widespread tendency of people to excuse a lack of success in mathematics because of a belief that it is a matter of inherent talent rather than lack of effort. Because of this, the National Mathematics Advisory Panel (2008) recommends that educational leaders help students and parents recognize the effect of effort on mathematics achievement.

Drawing on social/situational knowledge can influence student motivation for learning math. Fillingim and Barlow (2010) suggest that teachers are responsible for enabling their students to become "confident, skilled, independent *doers* of mathematics beyond our classrooms." Some students perform well in math because they recognize the shared expectations that the teacher and other students have, and then feel rewarded and fulfilled if they meet those obligations. Students who respond *only* to this "*normative identity*" of the classroom are motivated by external situations (Cobb, Gresalfi, and Hodge 2009). Although these students may be successful in the classroom environment, the mathematical skills and behaviors they have learned have no real meaning for them. Therefore, outside the classroom, the mathematical skills and behaviors they use in the classroom are seldom applied and are at risk of being forgotten.

In contrast, students become doers of mathematics beyond the limited environment of the classroom when teachers support learning experiences that align with the Process

Standards (NCTM 2000) and encourage a transition from external to internal motivation. These students begin to act in response to a "*personal identity*." The system of mathematical beliefs they develop is instilled in them and "becomes less what they *do* and more who they *are* as doers of mathematics" (Fillingim and Barlow 2010; Cobb, Gresalfi, and Hodge 2009). These students truly develop a productive disposition toward mathematics.

Teaching for Comprehension

As the "reading wars" raged in the 1990s, Keene and Zimmermann gave educators a dose of common sense with their book *Mosaic of Thought* (1997). In this remarkable text, the authors share their belief that learning to read depends on two critical factors—the teacher must fully understand the process of reading and must be determined to understand and respond to each child's individual needs as a reader. Teachers, torn between those advocating phonics-based instruction and those who favored a whole-language approach, read this book and breathed sighs of relief.

Moreover, with the intention of helping teachers "infuse in-depth, explicit, meaningful instruction in reading into literature-rich classrooms where teachers and children explore, learn from, and react to a wide variety of books" (Keene and Zimmermann 1997), these authors describe specific cognitive strategies that students can learn and use to enhance their ability to construct meaning from text.

Many of these strategies had been identified earlier in reading research (Pearson et al. 1992), but Keene and Zimmermann transport readers into classrooms sharing scenarios that describe explicit comprehension instruction. Teachers striving to produce avid, capable readers eagerly embraced this approach to teaching reading.

In the second edition of *Mosaic of Thought* (2007), the authors begin to link comprehension strategies to other disciplines with a document in the appendix, "Thinking Strategies Used by Proficient Learners." Readers, writers, mathematicians, and researchers who apply these *thinking* strategies move beyond simply comprehending what they read. As such, they provide a valuable toolbox for both mathematics and reading.

The Seven Comprehension Strategies

In compiling the strategies good readers use, Keene and Zimmermann (2007) drew six from the research of Pearson et al (1992) and added a seventh strategy—*monitoring meaning* (figure 1.2).

Fig. 1.2. Comprehension Strategies

1. Making connections—using schema and building background knowledge
2. Asking questions—generating questions before, during, and after reading to clarify understanding
3. Visualizing—using sensory and emotional images to deepen and expand meaning
4. Making inferences—using background knowledge with new information to predict, conclude, make judgments, or interpret

5. Determining importance—deciding what information is significant
6. Synthesizing—creating new ideas or extending/revising understanding based on engagement with texts or mathematic observations/investigations
7. Monitoring meaning—thinking about the degree of understanding and taking steps to improve understanding when necessary

(Adapted from Keene and Zimmermann 2007)

Anyone who works closely with young learners knows that when students feel competent and successful with a given activity, their interest in it soars. Reading teachers are aware that confident readers tend to read more, and the more they read, the more their skill increases. As a result, their confidence increases, prompting them to read even more. Unfortunately, students who lack confidence are likely to avoid reading. Once this occurs, the ability gap between the confident readers and those who lack confidence grows over time. Therefore, teaching all students how to become confident readers may prevent, or at least minimize, achievement gaps. And, according to Brassell and Rasinski (2008), "To become confident readers who readily comprehend, students need to have comprehension strategies. Teachers need to teach them these strategies."

Similarly, students who are confident of their mathematical abilities are much more willing to tackle problems, communicate mathematically with others, and think critically about math-related ideas. And, the more they engage in mathematical activities, the more their mathematical skills improve. Just as having a toolbox of comprehension strategies for reading gives students confidence and improves their ability to read, these tools, when adapted for mathematics, have the same effect. Teachers can help their students recognize the interdisciplinary nature of the reading strategies they are already using and encourage them to use the same strategies to improve their understanding of mathematics.

It is interesting to note that Pólya (1957) in his text *How to Solve It*, listed four steps, the first of which was "understand the problem." In more recent instructional materials, the first step has been revised and simplified to "understand the question" (O'Connell 2007). Students may understand the question, but still be stymied in their problem-solving efforts because they struggle with understanding the entire context of the problem. The comprehension strategies addressed in this book are powerful approaches that students can apply to help them understand these problems.

This book focuses on each of the comprehension strategies suggested by Keene and Zimmermann. In addition, Chapter 2 explores the importance of helping students increase their understanding of and ability to use mathematical vocabulary accurately, and offers suggestions for promoting vocabulary development in relevant mathematical contexts. Taking the literacy/mathematics connection even further, each chapter offers suggestions for incorporating children's literature into math lessons. The infusion of these texts piques student interest, makes the relevance of mathematics in daily life apparent, and creates a bridge between reading and mathematics.

Explicit Instruction

Most of us have had experiences in which we suddenly become aware of something that we had never before noticed, but that was right there in front of us. Until it was brought to our attention, we were completely unaware that it existed. Once it became obvious to us, however, it was hard to believe that we could ever have been unaware of it. After all, it was right there.

Remembering experiences like that help us understand the need to explicitly teach our students the comprehension strategies that we use regularly for reading and for mathematical thinking. Since these strategies are used by experienced readers and mathematicians with such ease, there is a tendency to expect them to be obvious to our students. Because they are not necessarily obvious, teaching the strategies explicitly opens the curtains for students, bringing this kind of thinking to their attention and providing valuable tools to enhance their learning.

What is meant by explicit teaching of these strategies? It is not an easy task "to make visible the unseen processes of creating meaning from text" (Murphy 2010). To do it well is one of the greatest challenges teachers face.

Explicit instruction can be broken down into six specific steps (Taylor et al. 1994):

1. Teacher explains *what* the strategy is.
2. Teacher explains *why* the strategy is important.
3. Teacher explains *when* to use the strategy.
4. Teacher *models how* to perform the strategy in an actual context while students observe.
5. Teacher *guides students* as they practice using the strategy.
6. Students *independently* use the strategy.

The emphasis is on *explicit* instruction because too often students are given a strategy (step 1) and are then expected to be able to use it independently (step 6). Without teaching the intermediate steps, students are unlikely to use the strategy. Teachers increase the success of their students when they provide clear explanations of not only the *what*, but also the *why*, *when*, and *how* followed by guided student practice and then finally independent student practice, gradually releasing responsibility to the students (Pearson and Gallagher 1983; Miller 2002). By following each of the six steps, teachers support their students throughout the learning process. Gradually, they withdraw support so students can successfully apply the strategies independently.

Explaining the "What"

Thought should be given to how to introduce a strategy. Before trying to explain a strategy to students, teachers should have a deep understanding of the strategy and how it is used effectively. Careful consideration should be given to the language used to define it. Is the explanation clearly worded and does the definition accurately convey what students need to know? Consistency in vocabulary usage is another consideration since establishing a common classroom language that can be applied across the curriculum is important. Furthermore, explanations should be concise—precisely expressing what the strategy is as well as what it is not.

Explaining the “Why”

The introduction of a strategy most often includes a description, but teachers frequently fail to explain why it is used. While the “why” may seem quite obvious, especially to experienced readers, teachers should be wary of assuming that it is obvious to students. A brief, explicit explanation about why using a specific strategy increases understanding ensures that all students are attuned to its utility.

Explaining “When”

Since it is often assumed that students know why a strategy is used, teachers may also assume that students know when to use it. Unless explicitly stated, students may mistakenly attempt to apply strategies at inappropriate times. This frequently occurs when young learners understand neither the “why” nor the “when.” By carefully reflecting on the use of the strategy and then clearly explaining it, teachers help their students identify when the strategy can be used effectively to increase understanding.

Modeling How to Perform the Strategy

Learners need to *see* and *hear* how strategies are used, so modeling and think-alouds are ideal instructional techniques for teaching students how to apply comprehension strategies to mathematics. During the modeling of strategy use, teachers “provide a verbal, declarative statement of how to do a skill or strategy” (Duffy 2003). They describe their thinking clearly and explicitly. Doing so, teachers “make the invisible visible and the implicit explicit” (Miller 2002).

Effectively modeling and thinking aloud how these strategies are applied may be difficult. Because the process of making meaning occurs so spontaneously for proficient readers, they rarely think about the strategies they use. Nevertheless, to assist students as they learn to apply comprehension strategies, it is essential that teachers take time to reflect on their own thinking as they read and then share that step-by-step process with their students.

Teachers are sometimes tempted to invite student participation during the modeling and think-aloud process. During the introduction of comprehension strategies, however, it is important that students focus on the teacher’s thinking, rather than being distracted by the comments of other students. Students should have plenty of opportunities to share their thinking in other contexts.

Effective teacher modeling consistently targets a specific strategy. It is very easy for a think-aloud to stray into other, related areas. If introducing several strategies during the think-aloud, the effectiveness of the modeling is diminished by blurring the distinctions between the different strategies. As students become proficient applying the strategies, additional modeling should be planned to show students how several strategies may be applied simultaneously when either reading or engaging in mathematical thinking.

Since everyone processes information differently, the modeling and think-alouds are only representations of the thinking one might do (Duffy 2003). Students should realize that the modeling is a guide as they implement the strategy, but that their own thinking is unique. The modeling of the strategy gives learners a “toehold” on the thinking involved, so that they can gradually make the strategy their own.

Guiding Students as They Practice

After they observe modeling and think-alouds by the teacher, students should practice applying the strategy with teacher support. Scaffolding by the teacher allows them to move just beyond where they would be able to go independently—the goal of the scaffolding is always the ability to use the strategies independently once the scaffolding is withdrawn (Vygotsky 1978). During this period of scaffolded practice, students begin to assume ownership of the strategies (Duffy 2003). Throughout the period of guided practice, teachers continually fine tune the support they provide based on observations and assessments of student work. Teachers gradually withdraw their support as students show that they can use the strategies independently.

The goal of providing scaffolding is teaching students to successfully apply comprehension strategies automatically and seamlessly (Harvey and Goudvis 2000). Keeping that in mind, students should be given ample opportunities to practice with targeted teacher support that provides cues when needed, but is gradually withdrawn as they become competent.

Giving Students Independent Practice

In the final step of explicit instruction, the responsibility for applying these strategies rests solely with the students. Teachers ensure that students have authentic opportunities to apply the newly mastered comprehension strategies. With this step, teachers informally assess understanding to verify that students have mastered the use of the strategy. Working with students in small groups or conferring with them one-on-one allows teachers to listen to students as they explain their thinking. These conversations with students are essential in accurately assessing their level of mastery and in identifying their future instructional needs.

Using Comprehension Strategies

In the explicit teaching of comprehension strategies, you must help learners know when to use specific strategies. Since comprehension of reading and mathematics is an ongoing process, a variety of strategies may be used appropriately before, during, and after reading or working with mathematical tasks (Pressley 2002; Brummer and Macceca 2008; Keene and Zimmermann 2007; Duffy 2003). Although listed separately in the following sections, the strategies overlap and supplement each other. They enhance understanding more thoroughly when used together than if used discretely. Model the integrated use of the strategies after students have had time to master the application of individual strategies to help students recognize how intimately they are interconnected.

“Beginning” Strategies

As students begin reading or engaging in mathematical activities, they should consider their goals. What do they hope to accomplish? What have they been asked to do? What do they need to do to be successful with their tasks? How will they know if they understand the concepts? These questions can be described as *monitoring meaning*.

Students may also be *making connections* they have with the text or task. Do they have personal connections? Have they experienced similar situations to those in the problem?

Have they ever used mathematics in the same way as it is applied in the math task or concept? Does it seem similar to solving other math tasks or concepts? Can students relate the problem to anything that they have read or heard before?

When students look at the title of a text, skim a math problem, or tinker with a math concept, they may predict what it is about. As they obtain new information from the text or task and combine that with what they already know, they begin to hypothesize about what to expect. They are *making inferences*.

Students may *ask questions*. What do they wonder about the topic or the task? What do they need to know to understand the reading or concept? What do they need to find out to solve the task? What questions may be answered by reading or understanding this concept?

"During" Strategies

In the midst of working with mathematics, any of the comprehension strategies may be appropriate for use to increase understanding and clarify thinking.

Students continue to *make connections* as they work. How can their experience or what they already know about the topic or task help them understand it better? What math concepts may relate to learning the task or concept? How have they solved similar math problems in the past?

As students pursue their mathematics work, they continue to *ask questions*. What else do they need to know to understand the concept more completely or to solve the problem? Is there any additional information that is required? Is there any extraneous information that should be ignored? Are there any patterns that can be observed? What is the author's purpose? (What is the instructional value of this task?)

Throughout the mathematics work, students should be *visualizing* or creating mental pictures. They create representations of the math concepts involved or act out a problem as they work on a solution. They may use concrete objects or represent their thinking with diagrams.

As they work, students continue to *make inferences*, drawing upon what they know from prior experiences and combining that with new information from their reading or task. What clues does the problem provide? What can I "read" between the lines? Reflecting on what I know, is my answer to this problem reasonable?

Students have to use their judgment to *determine the importance* of information in texts or math tasks. What is the main idea? What are the most important facts? What is the problem asking? Are there any parts that are irrelevant?

As they work, students *synthesize* the information and extend the meaning of the mathematics text or concept. What conjectures can be made from the patterns observed? How can this be applied to real-life situations? They also look critically at the text or the mathematics as they analyze and evaluate its validity. How well-founded is their reasoning? Are the collections of data reliable? Are there any built-in biases in the mathematical materials?

And, with any mathematical work, it is imperative that students actively *monitor meaning*.

Does the reading or solution make sense? If not, what can I do to make sense of it? Do I need to reread? Have I gone back over my work to check for accuracy? Can I explain my understanding?

“After” Strategies

The value of applying comprehension strategies does not end when the initial mathematical reading or work is complete. Good thinking requires reflection. Intentional use of comprehension strategies supports and strengthens the process of reflection. Application of these strategies may be even more effective when students work together to share their thoughts. The strategies listed below are especially appropriate as students look back and reflect.

Students may *make* further *connections* after reading or working with a math concept. How does this relate to other ideas or concepts? What experiences have I had that connect to what I just learned?

Sometimes, during the process of reading about mathematical concepts or problems to be solved, the overall picture is obscured by the details. Students should be encouraged to think back to *determine importance*. What was the most important part of what I read? What are the key aspects of the math concept? Why was solving the math problem a worthwhile task?

Having completed some reading or work with a math problem, it is an excellent time to *synthesize information*. How does everything fit together? What new ideas occurred to me using the information I learned? What conclusions did I reach? Did this change my thinking? How can I apply this to real-life situations? Is there anything that makes me question the validity of this information?

As students reflect, they should continue to *monitor meaning*. Did the reading or the concept make sense? Was the answer reasonable? Do I need to revisit my work to clarify my thinking?

Comprehension Strategies for Conceptual Understanding

When introducing a mathematical concept or problem that makes use of comprehension strategies, consider the four C’s: Conception, Connection, Construction, and Comprehension.

- **Conception:** Think about the concept or problem to be introduced. What are the foundations on which it builds? Where will students likely have difficulty? What vocabulary should be introduced? Which comprehension strategy will be most effective to help students develop a deep conceptual understanding?
- **Connection:** Consider the prior knowledge of students. How does the new concept or problem connect with their previous mathematical experiences? How might it connect with experiences they have had outside of school? How might it connect with their interests? When planning how to teach the concept, build upon these connections.

- **Construction:** Allow students to construct the meaning of the concept or problem by introducing important vocabulary, making explicit connections to students' prior experiences, providing hands-on learning opportunities, involving students in problem-solving situations, and encouraging conversations about the math they are learning. Model the use of the comprehension strategies and guide students as they apply them. Use both the "before" and "during" comprehension strategies.
- **Comprehension:** Continuously assess students' comprehension of the concept or problem and adjust instruction to respond to student needs. After mastery, include a reflection activity. This may be conducted with the whole class, in small groups, or in one-on-one conferences, although it tends to be most effective when students are able to share their thoughts with their peers. The ideas of one student often spur further thinking by other students. Ask students to record their thinking in math journals. During this period, have students use the "after" comprehension strategies.

Teaching Comprehension Strategies for Mathematics

Keene and Zimmerman's gradual release planning template for teaching reading comprehension strategies (2007) is easily adapted to mathematics. The template includes four phases: *planning phase, early phase, middle phase,* and *late phase.*

Planning Phase

- Identify the strategy to teach and explore its use in a mathematics context with grade-level colleagues.
- Plan mathematics experiences for students who are conducive to teacher modeling and think-alouds and who may be supported by the use of the identified strategy.
- Share with parents how the strategy relates to mathematics and encourage parents to discuss the strategy at home.

Early Phase

Instructional Focus:

- Think aloud about how mathematicians use the strategy.
- Discuss how they use the strategy when working with mathematical concepts and problems.
- Share how the strategy helps build understanding and aids in problem-solving.
- Model use of the strategy in diverse mathematical contexts.
- Model how to communicate mathematical thinking, both orally and in writing.
- Guide a Math Huddle—student mathematical discussion—where students

work together to apply the strategy; record the process on anchor charts for reference.

- In small groups and conferences, encourage students to articulate their thinking and use of the strategy.

Student Focus:

- Students experiment with the strategy individually, in groups, or as part of a Math Huddle.
- Students share their mathematical thinking through sticky notes, diagrams, math journals, and/or conversations.

Middle Phase

Instructional Focus:

- Think aloud and describe the use of the strategy with increasing complexity.
- Show how to apply the strategy in various contexts.
- Model and think aloud to show how the use of the strategy helps them understand more deeply and permanently.
- Share your mathematical thinking using mathematical vocabulary (orally, in writing, and in diagrams).
- Discuss ways in which the strategy relates to previously studied strategies.
- Confer with students to identify further teaching points and (informally) assess their abilities to use the strategy effectively.
- Create and meet with homogenous groups to provide instruction about using the strategy based on student needs.
- Keep parents informed of student progress.

Student Focus:

- Students apply the strategy in diverse mathematical contexts.
- Students express their mathematical strategic thinking in words (orally and in writing) and in diagrams.
- Students apply the strategy in progressively more difficult mathematical contexts.
- Students show evidence of applying the strategy independently.
- Students explain how they use the strategy and how it improves their understanding.
- Students share their use of the strategy with others to further comprehension.

Late Phase

Instructional Focus:

- Model and think aloud using the strategy in challenging contexts with small homogenous groups (needs-based).
- Demonstrate using the strategy to solve problems or to understand mathematical concepts in unfamiliar contexts.
- Share ways in which the strategy integrates with strategies previously learned.
- Begin the planning phase for the next strategy.

Student Focus:

- Students explain their use of the strategy clearly in conferences and to other students.
- Students apply the strategy and can accurately record their thinking in words (orally and in writing) and in diagrams.
- Students share how they use the strategy independently without prompting from the teacher.
- Students use the strategy in more challenging contexts.
- Students use the strategy flexibly.
- Students appropriately “mix and match” an effective use of strategies according to mathematical contexts.

Chapter Snapshot

Both reading and mathematics require purposeful thinking for the construction of meaning. They both involve teasing out big ideas from a set of discrete words, symbols, or procedures. Readers and mathematicians must be able to generate ideas, to express ideas clearly and with precision, and to justify their thinking to others (Carpenter, Franke, and Levi 2003). Naturally, the same comprehension strategies are effective when applied in either of the disciplines.

These strategies are, however, only a tool for facilitating and extending comprehension—the objective is to improve mathematical comprehension. Unfortunately, when comprehension strategies are taught in some classrooms, students are required to spend large amounts of time learning and practicing the strategies without really knowing the "how to" of applying them to increase understanding. The focus of instruction is the strategies themselves rather than building understanding. This misguided focus not only fails to help students increase their understanding, but also makes reading or mathematics work such a ponderous task that it becomes something students try to avoid.

The four planning phases outlined in this chapter offer guidance on how to teach the comprehension strategies, from modeling and think-alouds, through the gradual release of responsibility to students. With instruction and practice, students learn to choose and then effectively apply strategies that increase their mathematical understanding. Both proficient readers and mathematicians routinely use a combination of strategies in a seamless process that is mainly unconscious (Routman 2003). Instilling that same capacity in students is the ultimate goal in comprehension strategies instruction.

Review and Reflect

1. Reflect on the processes and strategies you use as you read to construct meaning. Which of these strategies do you also apply when working with math concepts and math problem solving? In what ways do you apply them?
2. In what ways can the use of comprehension strategies help your students develop a deep conceptual understanding of mathematics? How can you incorporate these strategies into your mathematics instruction?