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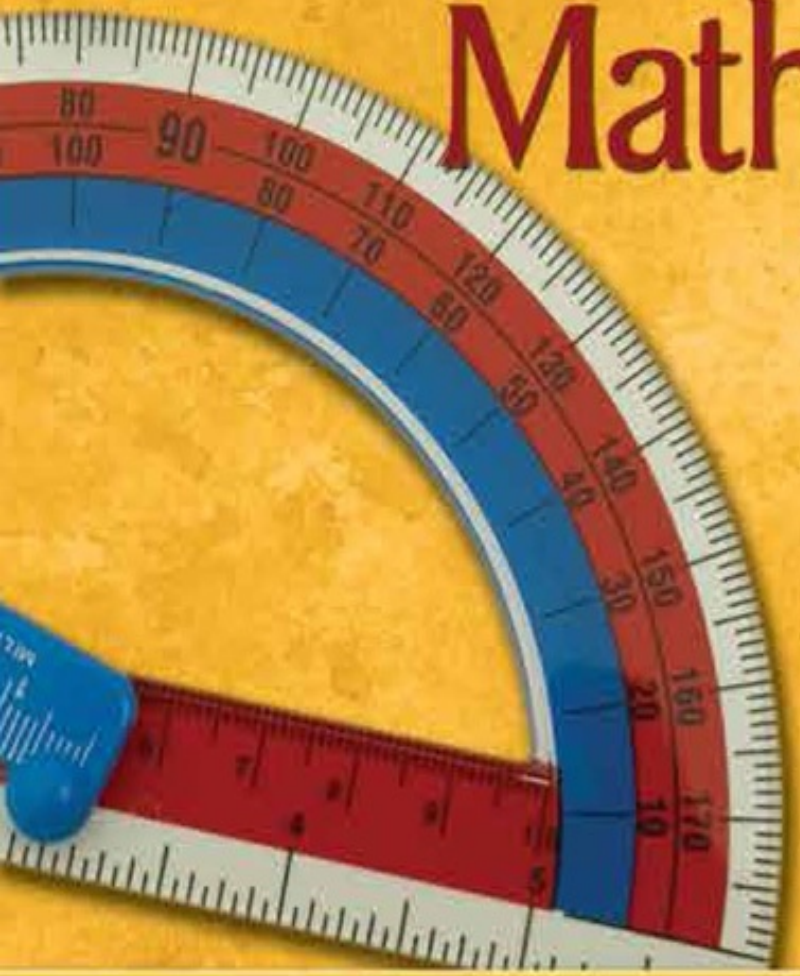
Professional Development for Successful Classrooms

Teaching Mathematics Today



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Teaching Mathematics Today

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Overview of Research

Teaching mathematics in today's diverse classrooms can be challenging, but it also provides teachers with many exciting opportunities to pass on life skills as well as mathematical knowledge. Mathematics is a subject that will directly affect every single student who enters the classroom. Proficiency in mathematics transcends simply succeeding in school or scoring well on state standardized tests. Mathematics teachers directly influence how students will approach problems and experiences in life: going to the grocery store, measuring walls to paint the house, borrowing money from a friend, cooking meals, driving a car, balancing checkbooks, paying taxes, buying houses, making investments, and so many other daily tasks.

There are many approaches to teaching mathematics. This professional development guide is built around a balanced approach to the instruction of mathematical concepts.

Common Approaches in Mathematics Instruction

Often teachers feel comfortable teaching the way they were taught. It is what they remember and what they know, so it becomes the way they teach, regardless of whether they believe it is the correct way to teach.

A common image of a "typical" mathematics classroom has the teacher standing at an overhead projector showing the equations and the formulas while the students take notes that will theoretically help them complete the assigned textbook problems. Many mathematics educators focus on skills and offer mostly procedural practice. Students learn the formulas and the procedures involved in the various mathematics disciplines. This form of instruction focuses on a lot of memorization and skill-anddrill practice. Teachers offer lecture type instruction and then students complete the pages in the texts during class time. Then they take home more practice worksheets for homework, with no further support for those who do not understand the mathematical procedures involved. This type of instruction is happening in

elementary schools as well as middle schools and high schools.

Textbooks are also a large part of a “typical” mathematics classroom. However, the use of textbooks alone can create a reduced use of effective instructional time because the textbooks often lack relevant guidance regarding how to address different learning styles, engage students, integrate manipulatives, and differentiate instruction based on the many individualized learning needs that mathematics instructors see in their classrooms. Often textbooks are filled with distracting pictures and designs that do not add to the mathematical comprehension of the key concepts. The textbooks also include many more topics than the average student can possibly learn in a school year. Teachers who lack guidance or experience may think they are supposed to open the textbook on day one and teach as far as they can by the end of the year. Therefore many educators are not teaching some of the important foundational concepts that students need in order to continue on to subsequent mathematics courses.

Another type of mathematics program leans more toward exploration of mathematical concepts through conceptual investigation. These programs are often very popular in elementary schools. Students use concrete materials, such as manipulatives, and participate in experiments and kinesthetic demonstrations that exhibit mathematical concepts. However, these types of lessons sometimes lack the connection bridging the “fun” activities to the actual mathematical concepts and abstract form of the problems. Sometimes this approach comes from a textbook-based program that offers little association between the learning activity with colorful pictures on one side of the textbook and the paper and pencil problems listed on the other side.

Often teachers are forced to follow strict district pacing charts or course outlines that delineate what concepts need to be covered during the school year. Because of the pressure teachers feel to cover all the topics, each topic is taught, practiced, and assessed and then the class moves on to the next topic. There is often little regard of whether students are actually able to truly learn and absorb each topic (Marzano, 2003). Consequently, only a subset of the students reaches a level of mastery of any given skill.

No one denies that there are indeed many students who succeed in these types of mathematics classrooms. They easily build on mathematical concepts and succeed in higher levels of mathematics courses. Yet there are also countless students who are struggling to achieve passing grades in their mathematics classes. Many students are failing district and state mathematics assessments. If teachers only explain the rules and evaluate correct or incorrect answers, then the students come away with a limited view of mathematical expertise (Lampert, 1990).

While no one method of instruction has been proven as the single best way to teach mathematics, using research-based designs and procedures have helped educators recognize the best features for approaching learning goals in mathematics (Hiebert and Grouws, 2007). Today's mathematics teacher needs to develop a scope and sequence that integrates multiple opportunities for practice prior to assessment, and time to reteach and revisit those skills that students have not mastered after the assessment.

A Balanced Approach in Mathematics Instruction

Research has established that students need both procedural and conceptual knowledge in order to learn and understand mathematics (NCTM, 2000). Knowledge of the procedures and formulas are critical to overall proficiency in mathematics. Also, exploration of the concepts through concrete experiments and manual manipulation of mathematical concepts is vital to the overall understanding of the "why" in mathematics instruction.

Therefore, a balanced approach can link these two distinct approaches and offer connections that can lead students to higher proficiency and understanding of mathematical concepts. It is ineffective to emphasize a high degree of procedural proficiency without developing conceptual knowledge. It is necessary to provide focused instruction that moves the student from the concrete to the abstract and then to the application of the concept (Marzano, 2003; Sutton and Krueger, 2002). Focusing on only the conceptual knowledge is not enough to help students achieve in the classroom and in real-world situations.

Teachers integrate alternative teaching methods, manipulatives, and additional practice into standard classroom lectures with a balanced approach. Teachers are given assistance in how to plan instruction so that lessons will align with state standards and address students' needs. Teachers employ strategies that address different learning styles, engage students, and differentiate instruction. There are multiple opportunities for practice prior to assessment, and time to reteach and revisit those skills that students have not mastered after the assessment. When students are given sufficient practice, they can approach being able to use the newly learned skill in new situations with accuracy so that that skill will be retained (Sousa, 2006).

A balanced approach calls for a change in the classic role of the teacher in a mathematics classroom. Teachers become facilitators, helping to move students' understanding of a given concept from the concrete to the abstract, and finally to the conceptual application of mathematical concepts. Research by Fillroy and Rolano has shown that the transition from concrete models to the algebraic equations can be difficult for students to achieve (Kieran, 1999).

Teachers need to be accessible to students and understand the process through which students need to progress in order for them to make the jump from the concrete phase to the conceptual phase. Following the concrete phase, students move to the abstract phase in which they learn the algorithm, using the understanding they gained from their experiences with the manipulatives. At this point, the conceptual understanding becomes important. It helps organize mathematical procedures and also helps students understand the appropriate procedures to apply in different situations (Kilpatrick, et al., 2001). Ultimately students need to move to a procedural fluency, which is "the ability to compute, calculate, and use rules and formulas correctly, quickly, and with assurance" (Dean and Florian, 2001). Without this, students will struggle to deepen comprehension of mathematical ideas and the ability to solve mathematics problems (Kilpatrick, et al., 2001).

Research suggests that the traditional dichotomies of explaining the best categories for teaching skill efficiency and conceptual understanding are no longer helpful. The features of teaching that facilitate skill efficiency and conceptual understanding do not fall neatly into categories frequently used to contrast methods of teaching, such as expository versus discovery, direct instruction versus inquiry-based teaching, student-centered versus teacher-centered teaching, and traditional versus reform-based teaching (Hiebert and Grouws, 2007).

Therefore, the balanced approach suggested in *Teaching Mathematics Today* crosses beyond these common method labels to create a collection of great strategies, sample charts for recording classroom management information, and assessment information for teachers to easily and effectively use to meet students' needs. The suggestions in this book will assist teachers in developing their own research-based, best teaching strategies to address different learning styles, engage students, and differentiate instruction.

Components of a Balanced Approach in Mathematics Instruction

Standards-Based Instruction

The National Council of Teachers of Mathematics (NCTM) believes that all students should learn important mathematical concepts and processes with understanding (NCTM, 2000). In an effort to help teachers meet higher standards and the diverse needs of students, the NCTM Board of Directors has formed several documents designed as aids to anyone making decisions regarding mathematics education of prekindergarten through grade 12 students. Originally there were three separate documents. One focused on curriculum, one on professional standards, and the last on assessment

standards. These represented the first attempt to give extensive mathematics-related goals for the educational field, and were well received. Over time, these documents were revisited, reviewed, and revised. The result was that in 2000, NCTM published one document to encompass all the goals for teaching mathematics in education. It is called *Principles and Standards for School Mathematics*. It serves as the basis for many states' mathematics standards and as support for decisions regarding mathematics in schools and what should be taught at each grade level. The teaching strategies for a balanced approach to mathematics instruction that are described in *Teaching Mathematics Today* address each of the six principles within the revised NCTM document: equity, curriculum, teaching, learning, technology and assessment. By aligning with standards, districts can work toward the NCTM challenge that everyone deserves to understand mathematics. It is not only for a select few (NCTM, 2000).

Integrated Curriculum

> Rather than working on subjects in isolation from one another, studying reading apart from writing, and apart from math, science, social studies, and other curricular areas, children learn best when they are engaged in inquiries that involve using language to learn, and that naturally incorporate content from a variety of subject areas. (NCTE, 1993)

It is important for students to understand that education is not a series of compartmentalized subjects that have nothing to do with one another. Rather, students need to realize that learning is more like a rug, where all subjects are woven together to create a broad scope of understanding that is ultimately most useful when all the strands fit together.

Teachers of mathematics must put concepts into real-life context for students in order for them to understand the concepts and make them personal. "When mathematics evolves naturally from problem situations that have meaning to children and are regularly related to their environment, it becomes relevant and helps children link their knowledge to many kinds of situations" (NCTM, 1989). This real-life context, which is necessary for developing student understanding, comes from integrating other subjects into mathematics instruction.

Language skills are most commonly and easily integrated into mathematics instruction. From an early age, a student is exposed to literature and develops a level comfort using and discussing books. "Opportunities for discourse in both reading and mathematics instruction promote children's oral language skills as well as their ability to think and communicate mathematically" (Moyer, 2000). Literature also provides a familiar context through which mathematical concepts, problem solving, patterns, and data can be explored and understood. Students do not see the need to learn *about* mathematics until they can learn about real life *through* mathematics.

The nature of inquiry embedded in science and social studies lends itself to the use of mathematics as a tool for understanding and extension. Mathematical concepts such as data collection, comparison, and analysis; patterns; probability; and graphical representations can all be learned and understood using science and social studies topics.

Technology is an ever-growing and changing field in education. It is important for mathematics teachers to help students understand that technology can be used for more than playing games and text messaging friends. Mathematical concepts can be enhanced and explored through the use of the Internet, computer software, graphing calculators, and other technology products.

Teaching Mathematics Today provides suggestions and strategies for teachers to integrate mathematics across the curriculum. It provides information about reaching all learners and broadening students' understanding of mathematical concepts.

Student Engagement

The strategies in *Teaching Mathematics Today* are geared toward engaging students and creating motivation for their learning processes. Cathy L. Seeley, 2004 president of the National Council of Teachers of Mathematics (NCTM), discussed in her message "Engagement as a Tool of Equity" how students' active engagement in their own learning impacts their achievements. She states, "Student engagement is perhaps our most important tool in our battle for equity." When students are actively motivated and busy reaching learning goals, they are also actively constructing knowledge and moving toward successful mastery of key concepts. The teacher is not the only indicator of student success in this model. Rather, the students have opportunities to have ownership and a greater understanding of the ideas and concepts that they are interacting with. When students are actively involved in writing, modeling, exploring, and discussing mathematics versus simply watching the teacher do these things, students are more likely to be successful (Seeley, 2004b). Using manipulatives, taking notes, presenting *PowerPoint* slide shows, and having students model problems are examples of strategies that actively engage students in the learning process. *Teaching Mathematics Today* presents various strategies for student engagement in mathematics lessons.

Differentiated Instruction

As students move from the concrete, to the abstract, to the application phase of learning, they are exposed to a concept or skill numerous times. Students should have multiple experiences with topics, allowing them to integrate the topics into their knowledge base (Marzano, 2003). However, not all students

process the new information in the same ways or bring the same skill sets to the learning experience. Some students need extra time to process concepts and look at problems in different ways (Sutton and Krueger, 2002). Other students need further teaching or teaching presented in multiple ways. *Teaching Mathematics Today* provides charts, strategies, and tips for identifying individual student needs and how to differentiate instruction to meet those needs within the classroom.

Cooperative Learning

Cooperative learning tasks are encouraged and described in this book. Cooperative learning offers the opportunity for students to learn from one another (Sutton and Krueger, 2002). In addition, students can be actively involved in their success and assume responsibility for their own learning with the support of other students. All students can benefit from cooperative learning. It enables high-performing students to stretch their understanding of a concept and make new connections between material. “Results are quite promising for using peer-assisted learning with low-performing students …” (Gersten and Clarke, 2007a). These students benefit from seeing the material presented in multiple ways where they can be actively involved in learning mathematical concepts. *Teaching Mathematics Today* provides teachers with an understanding of how to incorporate this method of instruction into the concepts and lessons they are already teaching.

Problem Solving

Research shows that students who are not successfully mastering mathematical concepts tend to demonstrate slow or inaccurate retrieval of basic mathematical facts, lean toward impulsivity when solving problems, and have difficulty forming mental representations of mathematical concepts or keeping information in working memories (Gersten and Clarke, 2007b). One study found that children improved in overall mathematical proficiency when they were taught mathematics through problem-solving strategies. Not only were they achieving better test scores, but also increasing in the ability to communicate their understanding of the mathematical concepts orally and in writing. The conclusions followed that the problem-solving approach to mathematics showed students and teachers that the two were connected and that the problem-solving strategies helped with overall mathematical proficiency (Hartweg and Heisler, 2007).

Research has shown that real-life applied activities and problem-solving activities establish a contextual setting for many lessons, providing motivation and encouraging curiosity (Hiebert and Carpenter, 1992). Overall, the challenging and interesting tasks found in application problems help teachers engage students in learning (Seeley, 2004a). Integrating problem

solving as one aspect of the curriculum follows the balanced approach of mathematics instruction.

Teaching Mathematics Today offers a step-by-step process to teach students problem-solving strategies. The set activity examples are meant to create independent, competent student problem solvers.

Guided Practice

In guided practice, teachers take a "we do" approach to help their students understand the concept being taught. The communication and interaction between the teacher and student have to be more significant than just solving problems on a worksheet. "Practice does not make perfect, it makes permanent" (Sousa, 2006). Through the guided-practice methods encouraged in this book, teachers monitor students' early practice and make sure it is accurate. They provide timely feedback so that the skills are learned permanently and correctly. Guided practice helps reduce initial errors and informs students of the critical steps in applying new skills (Sousa, 2006). As already mentioned, *Teaching Mathematics Today* incorporates instructional strategies that balance procedural proficiency and conceptual understanding, while actively engaging students in practice experiences that are designed to deepen their understanding and connect their mathematical knowledge to real life.

Manipulatives, Games, and Calculators

Manipulatives are essential to helping students understand mathematical concepts. Using manipulatives regularly provides hands-on experience and helps students construct useful meanings for the mathematical concepts they are learning (Grouws and Cebulla, 2000).

The use of manipulatives has become common in the primary grades and has proven to be an effective tool for illustrating elementary mathematical concepts. When students use concrete objects to represent mathematical ideas, they learn to organize their thinking and reflect on concrete representations (Dean and Florian, 2001). These same tools can be very effective in middle and high school mathematics classrooms. For example, manipulatives, such as algebra tiles, have extended this physical representation into Algebra I and provide a basis for developing algebraic concepts (Sharp, 1995).

Students need ample opportunities to practice in order to be able to execute procedures automatically without conscious thought (Kilpatrick, et al., 2001). Playing games with the goal of reinforcing skills, rehearsing information, and building retention of mathematical concepts is one way to allow students the practice time necessary for a skill to be mastered. Students in today's classrooms are very motivated by entertainment. Mathematical games can

pique their interests and give them a sense of fun while they are learning.

The use of graphing calculators can be helpful as well. According to research (1997) compiled by Dr. Bert Waits, cofounder of the developmental program Teaching with Technology, and Heidi Pomerantz, a professor of mathematics at Ohio State University, “graphing calculators can improve classroom dynamics, boost students’ confidence levels, and promote understanding of mathematical concepts and functions.” Further research shows that students who had access to calculators were better with mental calculations and estimations, as well as better able to solve real-life problems. Student achievement in general was higher (Kilpatrick, et al., 2001; Heller, et al., 2006). With increased use of graphing calculators during instruction, higher test scores were achieved even if students did not have access to the graphing calculators during the test (Heller, et al., 2006).

Teaching Mathematics Today discusses how manipulatives, games, and calculators serve as incredible tools for engaging students as well as addressing the needs of kinesthetic, visual, and English language learners. This book provides management techniques and strategies for using these materials in the classroom.

Vocabulary Development

Teaching Mathematics Today also provides teachers with resources for developing the specialized vocabulary necessary for mathematical-concept comprehension. Mathematical language is very precise compared with the English used in common discourse. This makes the study of mathematical vocabulary different from most other content areas students study. Various vocabulary-development activities are available so that students, including English language learners, can truly understand the academic vocabulary that will help them unlock the mathematical concepts.

Intervention

Rather than waiting to find out which students will require intervention and additional instruction in order to pass the required mathematics classes, there is an increasing need for innovative programs that prepare students to comprehend mathematical concepts and fill in achievement gaps. “We must expect all of our students to learn mathematics well beyond what we previously expected. We need all students to be more proficient than in the past, and we need many more students to pursue careers based on mathematics and science” (Seeley, 2005). In order to reach these goals, mathematics programs are needed that offer foundational concepts with a balance of computational and procedural skills, conceptual comprehension, and problem-solving practice so that students can build on general

mathematics proficiency. Effective intervention programs that prepare students for college and higher education should focus on readiness rather than just remediation (Oesterreich, 2000). Some of the most effective teaching practices suggested for low-achieving students as well as special education students are visual and graphic depictions of problems, systematic and explicit instruction, small-group instruction, student think-alouds, peer-assisted learning activities, and formative assessment data (Gersten and Clarke, 2007a). These teaching practices are infused in many of the strategies offered in this book.

Assessment and Data-Driven Instruction

Standards-based instruction begins with the goal of all students mastering the given curriculum with appropriate instruction, materials, and support. In order for this goal to be achieved, teachers must have a firm grasp of where students are in their process of learning a mathematical concept, what they need to accomplish to achieve mastery, and how they will reach the set goals (Wiliam, 2007). Teachers must then use formal and informal assessment strategies "minute by minute and day by day, to adjust their instruction to meet their students' learning needs" (Wiliam, 2007). Assessments provide teachers with the necessary data to understand which students are struggling in specific areas of the curriculum.

This book provides strategies and charts for formal and informal assessments, as well as ways to use data to drive further instruction within the classroom.

The National Council of Teachers of Mathematics Standards/Focal Points

> The Curriculum Focal Points are the most important mathematical topics for each grade level. They comprise related ideas, concepts, skills, and procedures that form the foundation for understanding and lasting learning (http://www.NCTM.org).

The National Council of Teachers of Mathematics (NCTM), one of the leading nationwide authorities in teaching mathematics, has been providing standards and suggestions for school mathematics courses for decades. The content standards found in the *Principles and Standards for School Mathematics* (NCTM, 2000) are used either directly or as a comparison standard for school districts across the nation. Many states refer to this guide as a model of how to develop and cultivate mathematics comprehension for students who progress through each grade level toward graduation from high school. These content standards are divided by grade bands—prekindergarten through second grade, third through fifth grade, sixth through eighth grade, and ninth through twelfth grade. Each grade band encompasses the various disciplines of mathematics: number and operations; geometry; measurement;

algebra; and data analysis, probability, and statistics.

Following the authoring of this comprehensive collection of standards, NCTM created the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. This document encompassed the major focus areas for which to provide emphasis for the included grade levels. The goal was to meet the needs of increasing accountability, students and teachers who move often, and the cost of continually developing mathematics curriculum. By pulling out only the most vital concepts that are necessary in that grade level, this document was created to describe baseline standards for student knowledge. In this document, there are descriptions of the mathematical concepts and skills, rather than lists of the goals, standards, objectives, and learning expectations, as are found in the standards document. The collection of descriptions was created to inspire teachers to discuss with one another the direction of the mathematics courses in the school. They were created to guide the formation of mathematics curriculum and to inspire the strategies and lesson plans used to teach mathematical concepts. “This work may assist in the creation and eventual development of new models for defining curriculum, organizing instruction, developing materials, and creating meaningful assessments that can help students learn critical mathematical skills, processes, and ways of thinking and can measure and communicate what students know about the mathematics that we expect them to learn” (NCTM, 2006).

How to Use This Book

Teaching Mathematics Today is meant to be a guide for mathematics teachers. The book is designed to span all the grade levels from kindergarten through grade 12, and also can be adapted for the various disciplines of mathematics.

- This book offers research-based explanations of the teaching strategies that are most critical and highly effective for mathematics teachers to include in teaching mathematical concepts.
- Each chapter focuses on a different aspect of teaching in a mathematics classroom.
- A school or mathematics department might choose to work through the entire book as they streamline and perfect their mathematics program.
- A school or mathematics department might choose to use this book as the basis for intervention programs being implemented.
- An individual teacher might choose to use the book to improve the

effectiveness of mathematics instruction in his or her classroom.

- Teachers could keep the book as a resource to refer to for the specific areas as they relate to a mathematics classroom.
- New teachers can read the extensive explanations of the strategies and employ them in their lessons.
- Veteran and new teachers can read and apply the techniques that are described in the chapters to their current instruction of mathematical concepts.
- Teachers can learn extensively about meeting the different needs and offering access to core curriculum for struggling students and English language learners.

Post-Reading Reflection

1. What was your definition of a balanced approach to mathematics instruction at the beginning of this chapter?

2. Would you revise your initial definition? If so, how?

3. Reflect on two components of a balanced approach in mathematics instruction and explain why these are important for teachers to understand.