



Mathematical Discourse

Let the **Kids Talk!**

Critique

Justify

Question

Barbara Blanke
Foreword by Steve Leinwand

Publishing Credits

Corinne Burton, M.A.Ed., *Publisher*; Conni Medina, M.A.Ed., *Managing Editor*;
Diana Kenney, M.A.Ed., NBCT, *Content Director*; Veronique Bos, *Creative Director*;
Robin Erickson, *Art Director*; Marissa Dunham, M.A., *Assistant Editor*;
Lee Aucoin, *Sr. Graphic Designer*

Image Credits

p.105 (top let) Courtesy of Jessic Djuic; p.105 (all others) Courtesy of Kimberly Kelly; p.117
Courtesy of Dr. Craig Froehle; p.134 (all) Courtesy of the Math Learning Center; all other images
from iStock and/or Shutterstock.

Standards

- © Copyright 2010. National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.
- © Copyright 2007–2017. Texas Education Association (TEA). All rights reserved.

Shell Education

A Division of Teacher Created Materials
5301 Oceanus Drive
Huntington Beach, CA 92649-1030
<http://www.tcmpub.com/shell-education>

ISBN 978-1-4258-1768-8

© 2018 Shell Educational Publishing, Inc.

The classroom teacher may reproduce copies of materials in this book for classroom use only. The reproduction of any part for an entire school or school system is strictly prohibited. No part of this publication may be transmitted, stored, or recorded in any form without written permission from the publisher.

Weblinks and URL addresses included in this book are public domain and may be subject to changes or alterations of content after publication by Shell Education. Shell Education does not take responsibility for the accuracy or future relevance and appropriateness of any web links or URL addresses included in this book after publication. Please contact us if you come across any inappropriate or inaccurate web links and URL addresses and we will correct them in future printings.

Table of Contents



Foreword	5
Acknowledgments	7
Introduction	9
Chapter 1	
What Is Mathematical Discourse?.....	13
Chapter 2	
Discourse and Mathematical Practice and Process Standards.....	25
Chapter 3	
Teacher Moves That Promote Effective Student Discourse	59
Chapter 4	
How Math Talks Promote Discourse: Arguments, Ideas, and Questions	91
Chapter 5	
Equity and Engagement.....	117
Chapter 6	
Getting Kids Ready to Talk! The First 20 Days of Discourse.....	133



Appendix

References Cited.....	169
Recommended Resources.....	177
5-Digit Magic Number Lesson Plan.....	180
Contents of the Digital Resources.....	182





Chapter 1

What Is Mathematical Discourse?

What does it mean to do mathematics, and why do so many of our students dislike learning it in the classroom? This is a question that pops up more often than it should. There is an uneasy resistance to math that boggles the educational mind and leaves us wondering why. Why such anxiety? One reason could be that math is seen by most students (and sometimes parents and guardians) as different from other subjects. Math is frequently taught as a performance subject, where getting to the correct solution is the only goal. They see the math that they do in the classroom setting as something disconnected from their world. For many, doing math in school is only that—doing. It means completing worksheets and getting answers, and rarely do students think about or desire to see the beauty of mathematics, to share ideas, to ask questions, to struggle productively, or to look for rich connections that can make math applicable to the real world. The nature of mathematics does not make it different, as many believe; instead, it is due to the widespread misconception that mathematics is a subject of rules and procedures, using strictly numbers, recalling facts quickly, and being fast to get right versus wrong answers. Teachers who believe in promoting mathematical discourse have seen students begin to change this baseline traditional view of mathematics and begin to see mathematics as a creative, visual, connected, and exciting subject. Students who engage in mathematical discourse are given this opportunity in many different ways. Ultimately, it is you, the teacher, who shapes mathematical learning for students you teach. The first step is to understand what mathematical discourse means.

Facilitating student engagement in mathematical discourse can be both revitalizing and demanding. It can be easy to get a conversation going by asking students to share their ideas or strategies to the class. Though their ideas often amaze us as educators,

mathematical discourse goes beyond students performing a show-and-tell of strategies and solutions. It's more than the class merely listening and clapping and then moving on. Knowing what to do with students' ideas and teaching them how to meaningfully ask challenging and clarifying questions during discussions can be quite critical. If done well, mathematical discourse can shape teachers' and students' perceptions of themselves as mathematicians, as well as their ideas about what being a mathematician involves.

According to Deborah Ball:

Discourse is used to highlight the ways in which knowledge is constructed and exchanged in classrooms. Who talks? About what? In what ways? What do people write down and why? What questions are important? Whose ideas and ways of knowing are accepted and whose are not? What makes an answer right or an idea true? What kinds of evidence are encouraged or accepted? (1991, 44)

NCTM (2000) defines mathematical discourse as both students' and teachers' ability to articulate mathematical ideas or procedures via talking, writing, asking questions, and responding to ideas. These are realized through various configurations in the classroom in Figure 1.1:

Figure 1.1 Student-Teacher Relationships in Mathematical Discourse

Definition	Explanation
Student-to-Teacher	The student primarily addresses the teacher even though the entire class or group hears the student's comments.
Student-to-Student	The student addresses another student.
Student-to-Group/Class	The student addresses a small group of students or the entire class.
Individual Reflection	The student documents his or her reflections about mathematics in writing.
Teacher-to-Student	The teacher addresses the student(s).
Teacher-to-Group/Class	The teacher addresses a small group of students or the entire class.

As we see, effective mathematical discourse is an interactive process. Students engage in various types of discourse at different cognitive levels, deepening their questions to lead to explanations and justifications that may be challenged, defended, or clarified. Students can thereby form new generalizations, initiating new conversations.

Most notably, it is the student who is in control of the conversation, and the teacher's role is that of discussion facilitator. It is these student-to-teacher questions, generated from conversations, which lead to explanations and justifications that may be challenged, clarified, and defended within the discourse classroom community. During discourse, students will often make new generalizations and conjectures that initiate a deeper level of student communication through the discussion of ideas, strategies, and thinking. These are some of the important things discourse provides: ways to scaffold mathematical thinking into the curriculum for *all* students. After all, it is the teacher who plays the crucial role in shaping discourse with signals to their students about how to value mathematical knowledge as well as ways to think about and know mathematics (Ball 1991; Blanke 2009).

It can be tricky to create and maintain discourse environments in the classroom for many teachers (Sherin 2002). Often, it's not enough to encourage students to discuss ideas and converse with each other. Teachers must also ensure discussions are mathematically productive and continually scaffold student academic growth. Asking teachers to use discourse requires teachers to develop a new sense of what it means to teach mathematics and to be effective, successful mathematics teachers (Sherin 2002; Smith 1996). At the same time, teachers must want to better understand their roles in developing discourse and make the choice to change to effectively implement mathematical discourse daily in their math classrooms. The choice to develop discourse communities can be nurtured through shared ideas and the desire to continually learn.

A Historical Perspective on Mathematical Discourse

We work in the fog of collective amnesia.

—Treisman 2016

Mathematical discourse is not a new idea! A careful study of mathematical history uncovers the fact that many ideas that have been called new are actually old, being revisited and reapplied into a broadened mindset. The examination of studies and research of the past has also contributed to current research of the effective usage of discourse in mathematics education. Unfortunately, there continues to be obstacles

in implementing research-informed instructional practices, such as discourse, in our classrooms. A brief look at the history of mathematical discourse points out that mathematics education continues to be cyclical. This lens gives insights into the factors that may still be in contention today, the influences that continue to alter instruction, and the motives and conflicts that shape the present day learning of mathematics.

Studying our knowledge of the past empowers us to speak with more authority and to make better-informed decisions about our everyday practices in the classroom. History can offer a better insight into the influence of mathematical understanding and mathematical discourse on student learning and classroom teaching. Realizing this vision of classroom discourse in mathematics education has proven to be challenging to implement over the years. This supports the importance of studying how teachers develop and sustain mathematical discourse communities, and it supports the goal of developing a deep mathematical understanding for all. It is a goal of this book to offer teachers ways to support mathematical discourse in their math classrooms and encourage all students to construct viable arguments with others because learning is a collaborative endeavor that emphasizes the importance of every student's depth of understanding.

As stated in NCTM's *Principles to Actions: Ensuring Mathematical Success for All* (2014), there are dominant cultural beliefs about teaching and learning mathematics that sometimes get in the way of implementing best practices, such as mathematical discourse, in our classrooms. Developments in the mid- to late-nineteenth century have profoundly influenced our knowledge with regard to how students learn and understand mathematics. The very first American school mathematics textbook was Nicolas Pike's *New and Complete System of Arithmetic—Composed for the Use of the Citizens of the United States* (1788). Pike's teaching process was 1) state a rule, 2) give an example, and 3) have students complete a set of practice exercises. This scripted approach established a method of delivering mathematics to students that was rigid and has become deeply rooted in educational culture. Many today still live in the shadow of these ideas and stereotypes about learning arithmetic that was articulated so long ago.

The first attempt to change instruction was offered by Colburn's (1849) discovery learning, which offered the idea for teachers to postpone student practice until after students had developed a deeper conceptual understanding of mathematics. This began the movement toward the use of discourse and talking about mathematical ideas. His idea was expanded upon again by Charles Davies (1850) and Edward

Brooks (1880). Davies and Brooks discussed how student understanding and thinking could be strengthened through the act of studying and talking about mathematics to cultivate one's mental abilities (Bidwell and Clason 1970). A backlash to Colburn's ideas was quick and reminiscent of today's math clashes. The outcry became that rules were necessary and students could not be expected to invent them. New publications proclaimed that they would satisfy parents who longed for arithmetic to be taught "the good old fashioned way" with concise and plain explanations of rules. Today, this is still somehow a serious roadblock for developing productive discourse in mathematics.

Secondly, arithmetic became formalized through the creation of a logical system of definitions, principles, and theorems. This led to mathematics being taught based on quantity, which allowed mathematical and psychological theory to blend. The belief was students learned mathematics from a quantitative reality via student perception and intuition. Through these ideas, knowledge was built through reasoning and imagination. William Brownell's *Meaning Theory of Learning* (NCTM 1935) proposed the ultimate purpose of arithmetic instruction is the development of the ability to think in quantitative situations. The word *think* is used cautiously; the ability to merely perform certain operations mechanically and automatically is not enough. Students must be able to analyze real or described quantitative situations (NCTM 1935, 28).

The third development in mathematical discourse is an emphasis on using manipulative objects in early number work to better understand mathematics. Toward the end of the nineteenth century, the impact of this "new" psychology on mathematics teaching was presented by William James in his book *The Principles of Psychology* (Boring 1957). James provoked an extensive reevaluation of what mathematical content should be taught and why. He spoke to the debate about the origin of the meaning of *number* by asking: Does number exist without any sensemaking by the user, or does meaning evolve from the human mind? (Bidwell and Clason 1970).

So, within the first half century of the founding of the United States, the great school of mathematics debate was underway. Should teachers give students rules and facts to memorize? Or, should they offer problems to talk, argue, discover, and develop a deep understanding of the underlying mathematical principles?

During the twentieth century, and extending into the present day, the teaching of mathematics in American schools has continued this debate and experienced identifiable phases, each with a different emphasis (see Figure 1.2 on page 18).

You've Just Finished your Free Sample

Enjoyed the preview?

Buy: <http://www.ebooks2go.com>